MANAGEMENT SCIENCE
Vol. 57, No. 10, October 2011, pp. 1811–1812
ISSN 0025-1909 | EISSN 1526-5501 | 11 | 5710 | 1811

http://dx.doi.org/10.1287/mnsc.1110.1394


# Note: A Reply to "New Product Diffusion Decisions Under Supply Constraints"

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In our prior work on product diffusions in presence of a capacity constraint, we postulated that a firm operating in such an environment should always attempt to fulfill as much of the present demand as is possible with the capacity constraint. In other words, the firm would never have demand backlogged while accumulating inventory. In this note, we derive a sufficient condition for the optimality of such fulfillment policy.

*Key words*: marketing–operation interface; Bass diffusion model; capacity planning
*History*: Received October 8, 2010; accepted May 26, 2011, by Candace A. Yano, operations management. Published online in *Articles in Advance* July 15, 2011.

## 1. Background

In Ho et al. (2002), we characterize the pattern of new product diffusion under a supply constraint. Proposition 1 of Ho et al. (2002) states that the firm must adopt the so-called immediate fulfillment policy (it is termed myopic selling policy in Ho et al. 2002). The immediate fulfillment policy states that it is always optimal for the firm to fulfill customer demand immediately.

Shen et al. (2011) recently developed a numerical example to show that the immediate fulfillment policy may be suboptimal in some new product diffusion settings. This example demonstrates that the stated conditions of Proposition 1 in Ho et al. (2002) are not sufficient to guarantee the optimality of the immediate fulfillment policy. To rationalize the optimality of the immediate fulfillment policy, Shen et al. (2011) develop a more general framework by allowing the firm to adjust (either increase or decrease) price over time. It appears that the immediate fulfillment policy is indeed optimal when *dynamic* pricing is allowed.

This note develops a sufficient condition for the optimality of immediate fulfillment policy under the *static* price scenario. The sufficient condition imposes a maximum rate at which waiting or backlogged customers can be lost. In this regard, our result complements that of Shen et al. (2011). Our result is useful when the firm does not have the freedom to adjust prices in the short-run because customers may develop peer-induced fairness concerns arising from different customers paying different prices for the same product (Ho and Su 2009).

Following the notation used in Ho et al. (2002), we use the following set of quantities to describe the new product diffusion dynamics for a make-to-stock firm with limited production capacity $c$:

- $D(t)$—cumulative demand for the new product at time $t$
- $S(t)$—cumulative sales of new product at time $t$
- $d(t)$—demand rate at time $t$, $d(t) = \mathrm{d}D/\mathrm{d}t$
- $s(t)$—sales rate at time $t$, $s(t) = \mathrm{d}S/\mathrm{d}t$
- $r(t)$—production rate at time $t$
- $W(t)$—customer backlog at time $t$
- $L(t)$—cumulative customer loss at time $t$
- $I(t)$—product inventory at time $t$

Kumar and Swaminathan (2003) and Shen et al. (2011) consider settings in which the firm's decision set includes both the production rate and the sales rate,[1] whereas in Ho et al. (2002), the production rate is preset as

$$r^M(t) = \begin{cases} c & t < t^*, \\ d(t) & t \geq t^*, \end{cases} \tag{1}$$

where

$$t^* = \min\left(t \mid d(t) < c, \frac{q}{m}s(t)(m - D(t)) - d\left(p + \frac{q}{m}S(t)\right) < 0\right), \tag{2}$$

and $p$ and $q$ are innovation and imitation parameters.

[1] Shen et al. (2011) also consider the profit margin trajectory as one of the controls.

The firm chooses the sales rate $s(t)$ to maximize profits for fixed values of capacity $c$ and launch time $t_l$:

$$
\begin{aligned}
P(c, t_l) &= \max_{s(t)\ge 0}\left(\int_0^T (as(t) - hI(t))e^{-\theta t}dt\right)\\
\text{s.t. } \frac{\mathrm{d}D}{\mathrm{d}t} &= d(t),\\
\frac{\mathrm{d}S}{\mathrm{d}t} &= s(t),\\
\frac{\mathrm{d}d}{\mathrm{d}t} &= \frac{q}{m}s(t)\,(m - D(t)) - d\left(p + \frac{q}{m}S(t)\right),\\
\frac{\mathrm{d}L}{\mathrm{d}t} &= lW(t),\\
\frac{\mathrm{d}W}{\mathrm{d}t} &= d(t) - s(t) - lW(t),\\
\frac{\mathrm{d}I}{\mathrm{d}t} &= r(t) - s(t),\\
L(t), I(t), W(t) &\ge 0,\\
D(0) &= S(0) = L(0) = W(0) = 0,\\
I(0) &= ct_l, \quad d(0) = pm,
\end{aligned}
\tag{3}
$$

where $a$ is the (constant) product profit margin, $\theta$ is the discount factor, $h$ is the inventory holding cost, and $l$ is the customer loss rate. We note that nonnegativity constraints on $I(t)$ and $W(t)$ imply that $r(t) \ge s(t)$ whenever $I(t) = 0$, and $d(t) \ge s(t)$ whenever $W(t) = 0$. In this formulation, to be consistent with Kumar and Swaminathan (2003) and Shen et al. (2011), we consider a finite planning horizon, defined by a sufficiently large $T$, e.g., $T \gg t^*$ (in Ho et al. 2002, the limit of $T \to \infty$ is used).

## 2. Optimality of the Immediate Fulfillment Policy

Consider the immediate fulfillment sales policy $s^M(t)$, defined as

$$
s^M(t) = \begin{cases} c & I^M(t) = 0,\ W^M(t) > 0,\\ \min(c, d^M(t)) & W^M(t) = I^M(t) = 0,\\ d^M(t) & I^M(t) > 0,\ W^M(t) = 0, \end{cases} \tag{4}
$$

where $d^M(t)$, $I^M(t)$, and $W^M(t)$ are the corresponding values of demand rate, inventory, and waiting pool size, respectively. The immediate fulfillment policy implies a sales rate of $c$ when inventory is zero and there exists a backlog, a sales rate that is capped by either $c$ or the demand rate (lower of the two values) when both backlog and inventory are zero, and a sales rate that is identical to demand rate when the inventory is positive and there exists no backlog. The following result states a sufficient condition for the immediate fulfillment policy under production plan (1) to be optimal.

Proposition 1. *For any launch time $t_l$, constant profit margin $a > 0$ and capacity $c$ in* (3), *the immediate fulfillment policy defined in* (4) *is optimal as long as*

$$
l \le l^M(\theta, p, q, m, c) = \frac{\theta(p+q)(c/m)\exp(-\frac{3}{2}(p+q)m/c)}{2(\theta+p+q)^2}. \tag{5}
$$

Proof. See the online appendix (available in the electronic companion).

Proposition 1 suggests that for a given set of problem parameters, the immediate fulfillment policy is optimal as long as the loss rate coefficient $l$ is not too high. In particular, in settings where customers are fully patient and unsatisfied orders are completely backlogged (i.e., $l = 0$), the immediate fulfillment policy is always optimal. The example developed by Shen et al. (2011) describes a setting in which customers are extremely impatient and all unsatisfied orders are lost ($l = \infty$), and here the immediate fulfillment policy may indeed no longer be optimal.

## 3. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

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